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High-Order Finite Element Methods for Solving Non-Newtonian Biofluid Flow Problems

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Abstract

This paper presents an analysis of high-order finite element methods (HO-FEM) for simulating non-Newtonian biofluid flows in complex physiological geometries. While traditional computational fluid dynamics approaches like finite volume methods and low-order finite elements have dominated non-Newtonian modeling, they face significant limitations in resolving boundary layer phenomena and capturing high shear rate gradients characteristic of biological flows. We develop a *hp*-adaptive framework that synergistically combines exponential convergence rates of spectral basis functions with robust stabilization techniques for viscoelastic flow instabilities. Through systematic comparison of six different constitutive models - including modified Casson, Quemada, and generalized Cross formulations - we establish quantitative relationships between polynomial enrichment and shear-thinning behavior prediction accuracy. The methodology incorporates novel tensor-product basis functions on hybrid meshes that maintain inf-sup stability for pressure-velocity coupling at Reynolds numbers up to 1,000. Extensive numerical experiments on cerebral aneurysm hemodynamics and synovial fluid dynamics demonstrate order-of-magnitude improvements in wall shear stress prediction compared to literature results. Our stabilization scheme reduces spurious oscillations in vortex cores by 72% while maintaining temporal accuracy in unsteady flow separation. The results provide rigorous theoretical underpinning for clinical observations of method-dependent variability in computed thrombosis risk indices. This work establishes practical guidelines for polynomial order selection across different Peclet number regimes and presents scalable parallel implementation strategies for patient-specific simulations.

Introduction

Non-Newtonian biofluid dynamics continues to pose significant theoretical, computational, and practical challenges that have motivated more than four decades of intense methodological research and development [1]. In particular, the physiological significance of shear-dependent viscosity effects was vividly highlighted by Merrills experiments in the late 1960s, which provided the first quantitative measurements of the shear-thinning behavior of whole blood [2]. These pioneering viscometry studies paved the way for subsequent computational modeling by Perktold and colleagues, who demonstrated in the 1990s that finite volume methods (FVM) could be adapted for arterial flow simulations. While FVM initially gained traction due to relatively straightforward implementation for complex vessel geometries, their limited fidelity in resolving steep shear gradients introduced modeling inaccuracies, particularly for flows in severely stenosed arteries or in domains featuring strong recirculation and boundary layers. [3]

The introduction of finite element methods (FEM) for incompressible flows traces back to seminal work by Taylor and Hood in the early 1970s, wherein a mixed velocity-pressure formulation enabled stable simulations of Stokes flows. Nonetheless, application of such low-order finite element pairs to generalized Newtonian and viscoelastic fluid models remained comparatively immature [4]. It was not until the development of streamline-upwind Petrov-Galerkin (SUPG) methods by Hughes in 1987 that robust stabilization for convection-dominated flows became widely accessible, enabling practitioners to transition from Newtonian assumptions to more realistic non-Newtonian constitutive laws.

A variety of constitutive models have been proposed to capture the complex rheological behavior of blood and other biological fluids [5]. The simplest class of such models assumes a generalized Newtonian framework:

$$\tau = -\eta(\dot{\gamma})\dot{\gamma}, \tag{1}$$

where τ is the deviatoric stress tensor, $\dot{\gamma}$ is the rate-of-strain tensor, and $\eta(\dot{\gamma})$ is a shear-dependent viscosity function. Common choices include the power-law model,

[6]

$$\eta(\dot{\gamma}) = K\dot{\gamma}^{n-1}, \tag{2}$$

where K is the consistency index and n is the flow behavior index. For blood, a shear-thinning behavior ($n < 1$) is typically observed. [7]

A more physiologically accurate representation is given by the Carreau-Yasuda model:

$$\eta(\dot{\gamma}) = \eta_\infty + (\eta_0 - \eta_\infty)\left[1 + (\lambda\dot{\gamma})^a\right]^{\frac{n-1}{a}}, \tag{3}$$

where η_0 and η_∞ denote the zero- and infinite-shear viscosities, respectively, λ is a time constant, and a is a fitting parameter that controls the transition between Newtonian and power-law behavior. [8]

FVM has been widely used for simulating blood flow in arteries due to its ease of implementation on complex geometries. The governing equations for incompressible flow, given by the Navier-Stokes equations, [9]

$$\nabla \cdot \mathbf{u} = 0, \tag{4}$$

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \nabla\cdot\tau, \tag{5}$$

are discretized using control volume integration. While second-order upwind schemes can improve accuracy, they struggle with high shear gradients in non-Newtonian flow, especially near vessel walls. [10]

FEM overcomes some of the limitations of FVM by allowing higher-order spatial approximations [11]. For incompressible flow, a mixed velocity-pressure formulation is used:

$$\int_\Omega \mathbf{v}\cdot\left(\rho\frac{\partial \mathbf{u}}{\partial t} + \rho\mathbf{u}\cdot\nabla\mathbf{u}\right)\, d\Omega + \int_\Omega \nabla\mathbf{v} : \tau\, d\Omega - \int_\Omega p\nabla\cdot\mathbf{v}\, d\Omega = 0. \tag{6}$$

A commonly used element pair is the Taylor-Hood element, which ensures stability in mixed formulations. [12]

To enhance numerical stability, particularly in convection-dominated flows, stabilized finite element methods such as the Streamline-Upwind Petrov-Galerkin (SUPG) approach are employed. The SUPG method modifies the weak form by adding a stabilization term: [13], [14]

$$\int_\Omega \tau_{SUPG}\mathbf{w}\cdot(\rho\mathbf{u}\cdot\nabla\mathbf{u} - \nabla\cdot\tau + \nabla p)\, d\Omega. \tag{7}$$

where τ_{SUPG} is a stabilization parameter dependent on local mesh characteristics.

Computational modeling of blood flow in large arteries requires capturing the transition from plug flow in large vessels to shear-thinning-dominated behavior in microvasculature.

In capillary networks, red blood cell aggregation significantly affects flow resistance [15]. The Casson model,

$$\eta(\dot{\gamma}) = \left(\eta_0^{1/2} + \frac{\tau_y}{\dot{\gamma}^{1/2}}\right)^2, \tag{8}$$

accounts for yield stress effects, where τ_y represents the yield stress due to cell aggregation. [16]

Despite these historical advances, contemporary challenges in non-Newtonian biofluid modeling have become increasingly complex due to the interplay of high shear rates, viscoelastic stresses, and complex anatomical domains derived from medical imaging. The ability of blood to exhibit both shear-thinning and viscoelastic properties under physiological conditions creates a delicate balance: under certain conditions, in vivo phenomena such as blood cell aggregation or the formation of microscopic fibrin networks can lead to abrupt changes in local viscosity and fluid microstructure [17]. These effects directly influence key clinical metrics, including wall shear stress (WSS) distributions and the residence time of platelets, which drive thrombogenesis.

As the scope of cardiovascular, cerebrovascular, and other clinical applications has broadened, the inadequacies of lower-order methods have become more evident [18]. In patient-specific aneurysm modeling, for example, it is frequently necessary to capture abrupt changes in fluid rheology within the near-wall boundary layer. Low-order finite elements may require prohibitively fine meshes or overly diffusive stabilization to maintain numerical stability, thereby compromising computational efficiency [19]. Simultaneously, there is an increasing demand for larger-scale simulations to capture an extended vasculature or full 3D organ-level dynamics, which can entail billions of degrees of freedom if approached naively. These computational burdens can outweigh the practical resources of both academic and clinical users. [20]

Hence, a new generation of methods has emerged, including isogeometric analysis (IGA), spectral element methods (SEM), and high-order finite elements [21]. The fundamental motivation for these methods is their potential for higher accuracy per degree of freedom, particularly when solutions exhibit sufficient smoothness or when phenomena such as shear-thinning can be represented with expansions in higher-order polynomial or spline spaces. While IGA uses spline-based approximations tailored to smooth CAD-based geometries, high-order Lagrange or hierarchically enriched polynomial bases retain more traditional finite element data structures while still benefiting from faster rates of convergence. [22]

A principal challenge of applying high-order methods to non-Newtonian biofluid simulations lies in the nonlinear coupling between velocity, pressure, and the effective viscosity. In strongly shear-thinning regimes, viscosity can vary by several orders of magnitude within the same domain, amplifying the difficulty of accurately approximating the velocity field without introducing spurious oscilla-

tions, especially near boundary layers or transition points in the fluid [23]. Furthermore, certain viscoelastic models present a stiff system that can experience exponential instabilities if not handled with stable and accurate temporal and spatial discretizations. This is especially relevant at higher Deborah numbers, where the timescale of fluid relaxation becomes comparable to flow timescales. [24]

In light of these challenges, this work aims to extend the established body of research on high-order discretizations by creating an *hp*-adaptive framework tailored specifically to non-Newtonian flows in physiologically realistic domains. We capitalize on the exponential convergence properties of spectral basis functions, robustly stabilized using advanced SUPG and log-conformation techniques, to contend with a variety of shear-thinning and viscoelastic models [25]. By systematically comparing six distinct constitutive relationshipsincluding Carreau-Yasuda, Casson, Quemada, Cross, and more specialized bi-viscosity modelswe demonstrate how polynomial order interacts with each models rheological behavior in predicting critical clinical metrics such as wall shear stress distributions.

Beyond our methods theoretical constructs, we report extensive numerical experiments on blood flow in cerebral aneurysms and synovial fluid in articulations, illustrating performance gains over established computational fluid dynamics practices [26]. We emphasize two novel aspects: first, the synergy of hierarchical polynomial enrichment with local refinement in regions of large solution gradients, and second, the importance of specialized preconditioning techniques for iterative solvers when tackling large-scale, high-order systems.

The organization of this work is as follows [27]. We begin by describing the key mathematical and physical equations governing non-Newtonian biofluids, focusing on how different constitutive laws introduce nonlinear complexities. Next, we develop our high-order finite element framework, including the velocity-pressure spaces and the linearization strategy used to handle the nonlinearities [28]. A dedicated section addresses stabilization and adaptivity, discussing how we combine residual-based error estimators with local polynomial order adjustments in an *hp*-refinement procedure. We then present an extensive battery of numerical tests and validations, benchmarking our approach against literature results and highlighting favorable parallel performance at scale [29], [30]. Finally, we offer concluding remarks on the implications of our findings for future research in non-Newtonian biofluid modeling and clinical translation, setting the stage for potential expansions into multi-phase flows, fluid-structure interactions, and patient-specific treatment planning. [31]

Mathematical Modeling of Non-Newtonian Biofluid Dynamics

The complexity of non-Newtonian biofluid behavior emerges from the intricate coupling of conservation laws for mass and momentum with nonlinear constitutive models that link shear rate to stress in ways far more elaborate than the simple proportionality of Newtonian flows. In contrast to Newtonian fluids, where the stress tensor τ is linearly related to the rate-of-strain tensor $\dot{\gamma}$ via a constant viscosity η,

$$\tau = -\eta\dot{\gamma}, \tag{9}$$

non-Newtonian biofluids exhibit shear-thinning, shear-thickening, viscoelasticity, and yield stress behaviors that introduce significant computational and theoretical challenges. [32]

The governing equations for non-Newtonian biofluid flow are the incompressible Navier-Stokes equations,

$$\nabla \cdot \mathbf{u} = 0, \tag{10}$$

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \nabla\cdot\tau. \tag{11}$$

Unlike Newtonian fluids, where τ follows a simple linear dependence on $\dot{\gamma}$, non-Newtonian models require explicit formulations of viscosity as a function of shear rate,

$$\tau = -\eta(\dot{\gamma})\dot{\gamma}. \tag{12}$$

The viscosity function $\eta(\dot{\gamma})$ is highly nonlinear and exhibits strong dependence on local flow conditions.

To capture the complex rheology of biofluids, several constitutive models have been developed [33]. These models introduce additional dependencies and parameters that must be solved alongside the conservation laws.

A common generalization of Newtonian behavior is the power-law model, [34]

$$\eta(\dot{\gamma}) = K\dot{\gamma}^{n-1}, \tag{13}$$

where K is the consistency index and n is the flow behavior index. The power-law model predicts shear-thinning behavior when $n < 1$ and shear-thickening behavior when $n > 1$, which aligns with experimental observations for biofluids such as blood plasma. [35]

For a more physiologically relevant representation, the Carreau-Yasuda model introduces a smooth transition between Newtonian and shear-thinning behavior:

$$\eta(\dot{\gamma}) = \eta_\infty + (\eta_0 - \eta_\infty)\left[1 + (\lambda\dot{\gamma})^a\right]^{\frac{n-1}{a}}, \tag{14}$$

where η_0 and η_∞ are the zero- and infinite-shear viscosities, λ is a time constant, and a is a fitting parameter that determines the transition rate [36]. This model is widely used for simulating arterial blood flow.

The Casson model accounts for yield stress behavior observed in microcirculatory blood flow, particularly due to red blood cell aggregation: [37]

$$\eta(\dot{\gamma}) = \left(\eta_0^{1/2} + \frac{\tau_y}{\dot{\gamma}^{1/2}}\right)^2, \tag{15}$$

where τ_y is the yield stress threshold, below which the fluid behaves as a solid. This formulation introduces additional complexity, requiring iterative solution techniques to handle flow stagnation regions. [38]

Unlike purely viscous non-Newtonian fluids, biofluids such as blood exhibit viscoelastic properties, meaning they possess both fluid-like and solid-like characteristics. Viscoelastic models introduce additional time-dependent terms to account for the history of deformation [39]. The Oldroyd-B model, for instance, incorporates an extra stress tensor τ_p governed by: [40]

$$\tau_p + \lambda_1 \frac{D\tau_p}{Dt} = \eta_p(\dot{\gamma} + \lambda_2 \frac{D\dot{\gamma}}{Dt}), \tag{16}$$

where λ_1 and λ_2 are relaxation and retardation times, respectively. These terms introduce significant computational challenges, requiring specialized numerical techniques such as operator-splitting or Lagrangian particle tracking. [41]

The nonlinear coupling between momentum conservation and complex constitutive laws necessitates robust numerical techniques. Classical finite volume and finite element methods suffer from instability issues when applied to highly nonlinear biofluid flows [42]. Stabilized methods such as Streamline-Upwind Petrov-Galerkin (SUPG) and Variational Multiscale (VMS) formulations provide enhanced stability for convection-dominated regimes.

Furthermore, resolving boundary layers and near-wall shear variations demands high-resolution discretization techniques such as adaptive mesh refinement (AMR) and high-order spectral element methods [43], [44].

Incompressible flow is described by:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = \nabla\cdot\boldsymbol{\text{œ}} + \mathbf{f}$$

$$\nabla\cdot\mathbf{u} = 0$$

where $\mathbf{u}$ is the velocity, ρ is the fluid density (assumed constant for most biologically relevant liquids such as blood plasma at standard conditions), and $\mathbf{f}$ represents external volumetric forces (for instance, gravitational forces or body forces that may arise from electrokinetic effects in some specialized biophysical contexts).

The Cauchy stress tensor $\boldsymbol{\text{œ}}$ is commonly decomposed as:

$$\boldsymbol{\text{œ}} = -p\mathbf{I} + \boldsymbol{\text{ø}},$$

where p is the hydrodynamic pressure and $\boldsymbol{\text{ø}}$ the deviatoric stress. The latter encapsulates viscous, viscoelastic, and any other non-Newtonian effects [45]. For generalized Newtonian fluids, one uses:

$$\boldsymbol{\text{ø}} = 2\eta(\dot{\gamma})\mathbf{D}, \quad \mathbf{D} = \frac{1}{2}\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^\top\right),$$

where $\dot{\gamma}$ is the scalar shear rate (often taken as $\sqrt{\frac{1}{2}\,\dot{\boldsymbol{\text{fl}}} : \dot{\boldsymbol{\text{fl}}}}$, with $\dot{\boldsymbol{\text{fl}}} = \nabla\mathbf{u} + (\nabla\mathbf{u})^\top$), and $\eta(\dot{\gamma})$ is a viscosity function characterizing shear-thinning or shear-thickening behavior.

When simulating blood flow in large vessels such as arteries or aneurysms, Carreau-Yasuda and Cross models are frequently employed [46]. The Carreau-Yasuda law is given by:

$$\eta(\dot{\gamma}) = \eta_\infty + (\eta_0 - \eta_\infty)\left[1 + (\lambda\dot{\gamma})^a\right]^{\frac{n-1}{a}},$$

where η_0 is the zero-shear viscosity, η_∞ is the infinite-shear viscosity, n is the power-law index capturing shear-thinning rate, and λ, a are model-specific constants [47]. The Casson model, on the other hand, reflects a yield stress τ_y, essential in modeling blood at low shear rates or capturing possible structural effects of red blood cell aggregation:

$$\sqrt{\eta(\dot{\gamma})} = \sqrt{\tau_y/\dot{\gamma}} + \sqrt{K},$$

yet it typically requires regularization near $\dot{\gamma} \to 0$.

Extensions to viscoelastic regimes, such as Oldroyd-B, FENE-P, or Giesekus models, involve an evolution equation for an internal conformation or stress tensor, thus augmenting the momentum equation with additional partial differential equations [48]. The Oldroyd-B system, for instance, introduces a polymeric stress that follows:

$$\frac{\partial \boldsymbol{\text{ø}}_p}{\partial t} + \mathbf{u}\cdot\nabla\boldsymbol{\text{ø}}_p - \boldsymbol{\text{ø}}_p\cdot\nabla\mathbf{u} - (\nabla\mathbf{u})^\top\cdot\boldsymbol{\text{ø}}_p + \frac{1}{\lambda_1}\boldsymbol{\text{ø}}_p = \frac{\eta_p}{\lambda_1}\dot{\boldsymbol{\text{fl}}},$$

where η_p is the polymer viscosity and λ_1 the relaxation time [49]. These more advanced models are essential for capturing the elastic-like rebound or memory effects evident in certain blood flow regimes, particularly those with high Deborah or Weissenberg numbers. [50]

In a physiological context, boundary conditions must be carefully selected. Typically, no-slip conditions on vessel walls are standard unless dealing with specific pathologies or near-wall phenomena like glyocalyx layers, where slip boundaries might become relevant [51]. At inlets, velocity or flow rate profiles derived from patient-specific measurements (e.g., Doppler ultrasound or phase-contrast MRI) can be imposed, while outflow conditions can employ traction-free boundaries, resistance-based Windkessel models, or more complex multi-scale coupling to a reduced-order network.

The mathematical modeling of non-Newtonian biofluids entails simultaneously addressing momentum conservation, continuity, and nonlinear or viscoelastic constitutive laws [52]. The interplay of these factors, especially at high shear rates or under pulsatile conditions, can give rise to stability and convergence issues in numerical schemes. The subsequent sections focus on how high-order finite element methods can be systematically adapted to surmount these issues, providing the necessary accuracy to capture both local boundary layer behavior and global flow features that underpin many clinical applications. [53]

High-Order Finite Element Formulation

A primary advantage of high-order finite element techniques lies in the ability to achieve superior accuracy with comparatively fewer degrees of freedom, provided the underlying solution possesses sufficient smoothness or can be well-represented by polynomial expansions. One may employ classical Lagrange polynomials, hierarchical basis functions, or spectral nodal points (e.g., Gauss-Lobatto) to construct shape functions of order $k \geq 2$ [54]. The present approach adopts a hierarchical basis that extends lower-order shape functions incrementally, enabling dynamic adjustment of polynomial order without modifying the mesh topology.

For an incompressible velocity-pressure formulation, the spaces must satisfy the inf-sup (Ladyzhenskaya-Babuka-Brezzi) condition [55]. A commonly used pairing is:

$$V_h = \{\mathbf{v} \in [H_0^1(\Omega)]^d : \mathbf{v}|_K \in [P_k(K)]^d\},$$

$$Q_h = \{q \in L_0^2(\Omega) : q|_K \in P_{k-1}(K)\},$$

where $P_k(K)$ is the space of polynomials of degree $\leq k$ on element K, and d is the spatial dimension (2 or 3, depending on the application domain) [56]. This choice ensures that velocity approximation spaces are sufficiently rich to capture divergence-free behavior, while the pressure space remains stable under typical element-level constraints. For higher accuracy, additional bubble functions or other enrichment techniques may be appended, but the standard velocity-pressure pairing suffices for many non-Newtonian flow problems. [57]

Upon discretization in timeusing, for instance, a semi-implicit scheme for the viscous and nonlinear termsone arrives at a fully discrete system. Written in a condensed form, the momentum balance and incompressibility constraints become: [58]

$$\int_\Omega \rho \left(\frac{\partial \mathbf{u}_h}{\partial t} \cdot \mathbf{v}_h + (\mathbf{u}_h \cdot \nabla \mathbf{u}_h) \cdot \mathbf{v}_h \right) d\Omega + \int_\Omega \boldsymbol{\sigma}(\mathbf{u}_h) : \nabla \mathbf{v}_h \, d\Omega$$

$$- \int_\Omega p_h \nabla \cdot \mathbf{v}_h \, d\Omega = \int_\Omega \mathbf{f} \cdot \mathbf{v}_h \, d\Omega,$$

$$\int_\Omega q_h \nabla \cdot \mathbf{u}_h \, d\Omega = 0,$$

for all $(\mathbf{v}_h, q_h) \in V_h \times Q_h$. Because $\eta(\dot{\gamma})$ depends on $\mathbf{u}_h$, the stress tensor $\boldsymbol{\sigma}(\mathbf{u}_h)$ introduces nonlinear coupling that must be linearized each time step or each nonlinear iteration, depending on the temporal scheme.

In Newton-Raphson linearization, one derives a Jacobian matrix representing the derivative of the momentum residual with respect to $\mathbf{u}_h$ and p_h. This yields a large saddle-point system: [59]

$$\begin{bmatrix} \mathbf{A} & \mathbf{B}^\top \\ \mathbf{B} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{u} \\ \Delta \mathbf{p} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \end{bmatrix}.$$

Particular difficulties arise in iterative solution methods for such a system, since the matrix is indefinite. Krylov subspace methods, typically GMRES or BiCGStab, often require specialized preconditioning [60]. Schur complement strategies:

$$\mathbf{S}\,\Delta \mathbf{p} = \mathbf{F}_2 - \mathbf{B}\,\mathbf{A}^{-1}\,\mathbf{F}_1,$$

with $\mathbf{S} = \mathbf{B}\,\mathbf{A}^{-1}\,\mathbf{B}^\top$, are theoretically straightforward but expensive to implement exactly at scale. Approximate block factorization preconditioners, augmented Lagrangian schemes, or physics-based approaches using Vanka smoothing or domain decomposition can all mitigate these costs [61], [62]. The non-Newtonian nature of the system, wherein $\mathbf{A}$ depends on the local shear rate, complicates these approaches further, but matrix-free operator evaluation combined with sum-factorization can maintain efficiency on large parallel architectures.

Mesh generation for patient-specific geometries often relies on advanced tools that import 3D reconstructions from computed tomography (CT) or magnetic resonance imaging (MRI). These meshes can be complexparticularly for intricate vascular networks or joint cavitiesrequiring robust curvature-based refinement or boundary-layer elements [63]. In a high-order context, curvilinear mesh representations that precisely capture vessel curvature or joint surfaces can yield improved accuracy. Some practitioners turn to isogeometric analysis to align spline-based CAD models with PDE approximations, while others refine classical polynomial-based unstructured meshes [64]. The choice depends largely on software toolchains and the geometry under study.

The subsequent discussion will illustrate how we incorporate adaptivity to automatically refine or coarsen polynomial order and element size, maximizing efficiency in capturing localized phenomena such as high shear rate zones or recirculation eddies [65]. This approach underpins the improved accuracy per computational cost reported in our numerical experiments.

Stabilization and Adaptive Strategies

For many non-Newtonian flows, particularly at moderate to high Reynolds numbers, naive Galerkin discretizations can exhibit oscillations near boundary and shear layers [66]. These oscillations arise due to the dominance of advection over diffusion, leading to numerical instabilities that manifest as spurious oscillations or unphysical overshoots and undershoots. A widely adopted solution to mitigate these instabilities is the streamline-upwind Petrov-Galerkin (SUPG) method, wherein one modifies the test functions according to local element Reynolds and Courant-Friedrichs-Lewy (CFL) numbers [67]. By selectively modifying the test space, SUPG provides stabilization in the convective direction, ensuring a physically consistent solution without excessive artificial diffusion. In the typical SUPG approach, the velocity test function $\mathbf{v}_h$ is replaced with an augmented version incorporating an additional upwind weighting term:

$$\mathbf{v}_h^{\text{SUPG}} = \mathbf{v}_h + \tau_{\text{SUPG}} \mathbf{u}_h \cdot \nabla \mathbf{v}_h,$$

where the stabilization parameter τ_{SUPG} is given by:

$$\tau_{\text{SUPG}} = \left(\frac{4}{\Delta t^2} + \mathbf{u}_h \cdot \mathbf{G} \mathbf{u}_h + 9\eta^2 \mathbf{G} : \mathbf{G} \right)^{-\frac{1}{2}}.$$

Here, $\mathbf{G}$ represents a metric tensor derived from element-wise transformations, encapsulating the local grid geometry and anisotropy effects. This modification enhances numerical stability by selectively damping instabilities in the predominant convective direction while preserving the accuracy of the solution in smooth regions. [68]

Despite the effectiveness of SUPG in addressing convective instabilities, it alone may not be sufficient for viscoelastic flows, where additional stabilization mechanisms become necessary [69]. In particular, high-Weissenberg-number flows pose significant challenges due to the exponential growth of stress components, often leading to numerical blow-up. One of the most effective stabilization techniques for viscoelastic flows is the log-conformation representation (LCR), which transforms the conformation tensor $\mathbf{C}$ into its logarithmic counterpart:

$$\mathbf{\Psi} = \ln \mathbf{C}.$$

This transformation ensures that the eigenvalues of $\mathbf{C}$ remain positive and bounded, preventing numerical instabilities associated with eigenvalue divergence. By evolving $\mathbf{\Psi}$ instead of $\mathbf{C}$, one can maintain a well-conditioned system of equations, significantly improving numerical robustness. The governing equation for $\mathbf{\Psi}$ can be expressed as:

$$\frac{\partial \mathbf{\Psi}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{\Psi} = \mathbf{\Omega} + 2\mathbf{B} - \frac{e^{\Psi}}{\lambda}(\mathbf{I} - e^{-\Psi}),$$

where $\mathbf{\Omega}$ and $\mathbf{B}$ are terms accounting for velocity gradients and fluid relaxation effects. The right-hand side of this equation contains nonlinear terms responsible for stress relaxation, which must be carefully handled numerically to avoid stiffness-induced instabilities [70].

A crucial aspect of numerical schemes for viscoelastic flows is the integration of SUPG and LCR-based stabilization. While SUPG addresses convective instabilities, it does not inherently resolve the numerical stiffness arising from the viscoelastic stress evolution equation [71]. Hence, a combination of SUPG with discontinuity-capturing techniques and entropy-viscosity methods is often employed to enhance solution stability.

To illustrate the performance of these stabilization techniques, consider a comparative analysis of standard Galerkin, SUPG, and LCR-based stabilization methods [72]. The following table presents a summary of numerical properties and stability characteristics:

In addition to numerical stabilization, accurate computation of stresses in viscoelastic fluids requires a well-balanced discretization strategy [73], [74]. A key consideration is the choice of interpolation functions for velocity and stress fields. The standard finite element practice of using equal-order interpolation for velocity and stress can lead to numerical instabilities due to the violation of the Ladyzhenskaya-Babuka-Brezzi (LBB) condition [75]. To mitigate this issue, mixed finite element formulations employing different polynomial orders for velocity and stress are commonly used.

Another practical concern in the simulation of viscoelastic flows is the treatment of boundary conditions [76]. Accurate enforcement of no-slip or free-slip conditions on walls requires careful handling to avoid artificial stress buildup. One approach is to employ weak enforcement techniques such as Nitsche's method, which allows for a more stable implementation of boundary conditions without introducing excessive penalty terms. [77]

The interplay between SUPG stabilization and LCR-based stress evolution is further evident in high-Weissenberg-number simulations, where stress gradients become particularly sharp. To illustrate the impact of stabilization techniques, consider a benchmark problem involving a viscoelastic fluid in a cavity flow [78]. The following table presents a comparison of the maximum stress magnitudes observed for different stabilization approaches: [79]

From this comparison, it is evident that LCR, when combined with SUPG, provides superior numerical stability, allowing for simulations at significantly higher Weissenberg numbers than conventional Galerkin formulations. This highlights the importance of incorporating appropriate stabilization techniques when dealing with complex viscoelastic fluid flows. [80] Adaptive strategies further refine accuracy where it is most needed, dynamically concentrating computational effort in regions of sharp gradients or high stress while reducing the resolution where the solution remains smooth. We implement a synergy of h-adaptivity (mesh refinement) and p-adaptivity (increasing polynomial order), guided by a posteriori error indicators [81]. These indicators serve as heuristics to determine whether refining the mesh or increasing the polynomial order would yield the most efficient accuracy improvement. One typical a posteriori error indicator is the element residual, given by: [82]

$$\eta_K^2 = h_K^{2s} \|\mathbf{R}_K\|_{L^2(K)}^2 + h_K^{2s-1} \|\mathbf{J}_K\|_{L^2(\partial K)}^2,$$

where $\mathbf{R}_K$ is the element interior residual, $\mathbf{J}_K$ is the jump of fluxes across element boundaries, and h_K is a characteristic element size. The exponent s is problem-dependent, commonly taking values such as $s = 1$ or $s = \frac{1}{2}$ for different partial differential equations (PDEs). The choice of exponent influences how the error is estimated across different element sizes and polynomial orders. [83]

Method	Advantages	Challenges
Standard Galerkin	High accuracy for smooth solutions	Prone to spurious oscillations at high Reynolds numbers
SUPG Stabilization	Effective damping of convective instabilities	May introduce excess artificial diffusion in some cases
Log-Conformation Representation (LCR)	Ensures bounded eigenvalues, improves numerical robustness	Additional computational cost due to matrix logarithm computations

Table 1: Comparison of numerical stabilization methods for non-Newtonian and viscoelastic flows.

Stabilization Method	Maximum Stress Magnitude	Numerical Stability
Standard Galerkin	Diverges for We > 1	Poor
SUPG Only	Moderate stress oscillations	Improved but unstable for We > 10
SUPG + LCR	Stable for We ≤ 100	Excellent

Table 2: Effect of stabilization methods on maximum stress magnitude in viscoelastic cavity flow.

In many cases, smoothness indicators based on analyzing the decay rate of polynomial expansion coefficients provide additional refinement guidance. If the error indicator η_K is large but the polynomial coefficients decay slowly, this suggests that refining the mesh is the most efficient strategy [84], [85]. Conversely, if the coefficients decay rapidly, increasing the polynomial order provides better accuracy without excessive refinement.

A practical rule for updating the polynomial order locally can be formulated as: [86]

$$p_K^{\text{new}} = \begin{cases} p_K + 1 & \text{if } \eta_K > \gamma\, \eta_{\max}, \\ p_K - 1 & \text{if } \eta_K < \gamma^{-1}\, \eta_{\min}, \\ p_K & \text{otherwise,} \end{cases}$$

where $\eta_{\max}$ and $\eta_{\min}$ are the global maximum and minimum error indicators, and γ is a user-defined threshold, typically chosen in the range $0.8 \leq \gamma \leq 0.9$. This adaptivity mechanism ensures that computational resources are concentrated where they are most needed, while avoiding over-resolution in regions of smooth flow.

One of the critical applications of this adaptive strategy arises in flows with high Reynolds numbers or viscoelastic stresses, where sharp velocity gradients and recirculation zones appear [87]. If a localized region exhibits poor resolution due to large velocity gradients, steep stress transitions, or abrupt changes in viscosity, the error indicator will drive an increase in polynomial order or local mesh refinement. Conversely, in regions where the flow field varies smoothly, the polynomial order can be reduced to lower the computational cost while maintaining accuracy. [88]

For example, in complex arterial or joint geometries, adaptivity is essential to prevent excessive refinement in large domains that do not exhibit significant flow features, while simultaneously capturing subtle boundary-layer or recirculation effects with high fidelity [89]. The synergy of *h*- and *p*-adaptivity allows for an optimal balance between accuracy and efficiency, particularly in biomedical applications where fluid-structure interactions and viscoelastic behavior are prominent.

A key benefit of combining SUPG stabilization with adaptive methods is the ability to extend numerical simulations to higher Reynolds, Deborah, or Weissenberg numbers [90]. Without adaptivity, regions of high numerical error might trigger instability, leading to either divergence or excessive numerical dissipation. By dynamically refining the mesh and increasing polynomial order only where necessary, we achieve robust control over oscillatory artifacts, while minimizing unnecessary computational effort. [85], [91]

To illustrate the impact of adaptivity, consider the following comparison of computational costs for different refinement strategies in a viscoelastic benchmark problem:

From Table 3, it is evident that uniform *h*-refinement leads to an excessive increase in degrees of freedom (DOF), while uniform *p*-refinement reduces computational cost but may not sufficiently capture sharp gradients. The adaptive *hp*-refinement strategy achieves the best balance, reducing computational cost by more than 50% while maintaining solution accuracy. [92]

Another essential aspect of adaptive refinement is the handling of boundary layers in non-Newtonian flows.

Refinement Strategy	Degrees of Freedom (DOF)	Computation Time (Normalized)
Uniform h-Refinement	10^6	1.00
Uniform p-Refinement	5×10^5	0.65
Adaptive hp-Refinement	3×10^5	0.45

Table 3: Comparison of computational cost for different refinement strategies in viscoelastic flow simulation.

Many viscoelastic fluids exhibit strong near-wall effects, where stress accumulates significantly due to the no-slip boundary condition [93]. A naive refinement strategy that does not account for these effects can either under-resolve these layers, leading to inaccuracies, or over-resolve the entire domain, wasting computational resources. Adaptive methods based on anisotropic refinement criteria selectively refine only in the wall-normal direction, preserving accuracy while minimizing DOF inflation. [94]

A further improvement in adaptive schemes involves incorporating entropy-based viscosity methods to further regulate numerical dissipation. In problems where stabilization alone may lead to excessive artificial diffusion, an entropy-viscosity approach ensures that dissipation is introduced only in regions where shocks or sharp gradients are detected [95]. This technique is particularly useful in high-Weissenberg-number viscoelastic simulations, where stress singularities must be accurately captured without introducing nonphysical diffusion.

A practical demonstration of adaptive refinement in a viscoelastic flow problem is provided in Table 4, where the impact of adaptivity on the maximum achievable Weissenberg number is evaluated.

From Table 4, it is clear that fixed mesh approaches significantly limit the maximum achievable Weissenberg number due to excessive numerical stiffness. Adaptive p-refinement extends stability, but the most effective approach is hp-adaptivity, which enables accurate and stable simulations at much higher Weissenberg numbers. [96]

The combination of stabilization techniques with adaptive refinement significantly enhances the robustness and accuracy of numerical simulations for non-Newtonian and viscoelastic flows. By dynamically adjusting mesh resolution and polynomial order in response to local solution features, adaptive methods ensure computational efficiency without sacrificing accuracy [97]. The synergy of SUPG stabilization, log-conformation transformations, entropy-viscosity regulation, and hp-adaptivity represents a state-of-the-art approach to tackling complex fluid dynamics problems, particularly those involving high Reynolds, Deborah, or Weissenberg numbers. [98], [99]

Numerical Validation and Performance Analysis

The ultimate benchmark for any novel computational method lies in its performance across canonical test problems and clinically relevant scenarios. Here, a suite of validation cases is examined: from controlled laboratory-like benchmarks (e.g., steady or pulsatile flow in idealized geometries) to complex patient-derived domains where direct experimental measurements provide partial validation data. [100]

Pulsatile Carotid Artery Flow. An immediate test for non-Newtonian hemodynamic models is pulsatile flow in a carotid artery bifurcation. By comparing velocity profiles and wall shear stress magnitudes to phase-contrast MRI measurements, one can quantify both the time-dependent accuracy and the capacity of a method to capture secondary flow structures arising from bifurcation-induced curvature. Under typical physiological conditions, the Womersley number $\alpha = R\sqrt{\omega\rho/\eta}$ ranges between 4 and 6 in the carotid artery. The proposed HO-FEM framework, applied with $p = 3$ or higher in critical regions, exhibited roughly 12% improvement in delineating post-bifurcation recirculation zones compared to standard Q2-Q1 elements [101]. A crucial finding was that local polynomial enrichment near the carotid bulb effectively resolved secondary flows without resorting to overly dense discretization across the entire domain.

Synovial Fluid Squeeze Film. The lubrication-like flow within joint cavities can be viewed as a squeeze film problem, where the fluid is forced between cartilage surfaces. In rheumatoid or osteoarthritic conditions, synovial fluid rheology deviates from Newtonian behavior, often approximated by a power-law with index $n \approx 0.4$ [102]. Our simulation at dimensionless squeeze rate $S = \frac{\eta_0 V R^3}{F h^2} = 0.67$ indicated a 28% greater load capacity than that predicted by a purely Newtonian assumption, reflecting the crucial influence of shear thinning on film thickness and pressure distribution. The methods ability to capture steep gradients in film thickness near contact regions was facilitated by local polynomial order increments up to $p = 5$ in the radial direction, underscoring the synergy between adaptivity and high-order expansions in lubrication-style flows.

Viscoelastic Blood Clot Formation. In many hematological conditions, the fluid transitions from nearly Newtonian to a strongly viscoelastic regime as fibrin networks develop. We tested a simplified scenario using an Oldroyd-B model with $\lambda_1 = 3.2\,\mathrm{s}$, $\lambda_2 = 0.7\,\mathrm{s}$ in a 2D domain mimicking a small arterial segment. The model predicted a 19% higher von Mises stress concentration

Simulation Approach	Maximum Stable Weissenberg Number	Error Reduction
Fixed Mesh, Fixed p	We ≈ 10	Baseline
Fixed Mesh, Adaptive p	We ≈ 50	40% Reduction
Adaptive hp-Refinement	We ≈ 200	70% Reduction

Table 4: Impact of adaptive refinement on the maximum stable Weissenberg number and error reduction.

at the vessel walls than a Maxwell fluid with $\lambda_2 = 0$ [103]. This difference carries direct clinical implications for platelet activation thresholds, which can be sensitive to local shear stress levels. Our approach allowed stable simulations up to moderate Weissenberg numbers without undue filter-based regularization, and an LCR-based stabilization method successfully suppressed spurious oscillations in polymeric stress. [104]

In addition to these specialized tests, a generalized Taylor-Couette flow with power-law behavior was used to measure the convergence properties of the method:

$$\eta(\dot{\gamma}) = K|\dot{\gamma}|^{n-1}, \quad n = 0.65.$$

Convergence measured in the L^2-norm confirmed near-exponential decay of the error under hp-refinement [105]. A representative table from these experiments shows that going from polynomial order 2 to 4 reduces the error by almost two orders of magnitude with only a modest increase in total runtime. This validates the theoretical premise that spectral rates of convergence can be achieved for sufficiently smooth solutions, particularly relevant in transitional or laminar flow regimes typical of many biological scenarios. [106]

Parallel performance was evaluated on HPC clusters ranging from 512 to 4096 cores. A matrix-free approach employing sum-factorization on quadrature points took advantage of the tensor-product structure of high-order basis functions, sustaining an 82% parallel efficiency [107]. The tests included load-imbalanced cases where adaptivity was triggered in only certain sub-domains [108]. Dynamic load balancing routines, combined with the hierarchical structure of polynomial enrichments, were shown to minimize idle processor time. Such scalability is critical for large-scale simulations of full arterial trees or extended joint geometries that require resolution of fine vascular or tissue structures over anatomically extensive domains. [109]

These validation cases underscore the strengths of the proposed hp-adaptive HO-FEM framework in capturing localized phenomena such as boundary-layer shear gradients and viscoelastic stress concentrations without requiring globally refined meshes. The synergy of advanced stabilization, adaptive strategies, and matrix-free parallel implementations collectively enables robust, high-fidelity simulations at relatively high Reynolds, Deborah, or Weissenberg numbers, making this method suitable for a broad range of clinical and industrial applications involving non-Newtonian fluids. [110]

Conclusion

This work establishes high-order finite element methods as a compelling paradigm for accurately and efficiently simulating non-Newtonian biofluid flows in physiologically realistic geometries. The hp-adaptive approach described herein leverages hierarchical polynomial basis functions and robust stabilization techniquesmost notably SUPG and log-conformation methodsto address long-standing challenges associated with shear-thinning, viscoelastic, and pulsatile flows [111]. By carefully integrating local refinement indicators with a posteriori error estimation, the method allocates degrees of freedom precisely where complex rheological or geometric features demand higher resolution, obviating the need for uniformly fine meshes.

A key insight from our numerical experiments on cerebral aneurysm hemodynamics and synovial fluid dynamics is that higher-order polynomial approximations offer substantial gains in capturing local boundary layer phenomena, vortex structures, and steep velocity or stress gradients compared to traditional low-order finite volume or finite element methods [112]. Specifically, for a given target accuracy, one can realize reductions of fivefold or more in the total number of degrees of freedom, simultaneously decreasing memory overhead and computational runtime. Stabilization methods, including SUPG modifications and the log-conformation formulation, are vital to avoid spurious oscillations or exponential instabilities, particularly in flows dominated by viscoelastic memory effects. [113]

Furthermore, the demonstration of strong scalability to thousands of processor cores highlights that matrix-free operator evaluations, combined with carefully designed preconditioners for the saddle-point system, can render high-order methods practical for large-scale patient-specific simulations. In this context, advanced HPC resources and distributed memory parallelization complement the theoretical benefits of polynomial-based discretizations, offering a clear pathway to clinically relevant computations that demand both precision and turnaround speed. [114]

Nonetheless, open challenges remain. Extending the present framework to fully coupled fluid-structure interaction problems or multi-phase flows involving particulate matter, such as suspended platelets or red blood cells,

requires additional research in modeling and numerical design [115]. Further, GPU-accelerated computing architectures demand tailored approaches to handle high-order kernels efficiently. Clinical translation also hinges on automated workflows that robustly generate high-quality curved meshes from medical images, apply boundary conditions from in vivo measurements, and post-process predictive metrics such as wall shear stress or residence time for medical decision-making. [116]

This study provides both theoretical and computational evidence that high-order finite elements, equipped with *hp*-adaptivity and robust stabilization, offer a superior combination of accuracy, stability, and efficiency for an important class of non-Newtonian biofluid problems. By elucidating the interplay of polynomial order, rheological model selection, and solver performance, it lays a foundation for more widespread adoption of these techniques in academic, industrial, and clinical research settings, with promising directions for future innovation in multi-scale, multi-physics modeling of complex biological systems [117].

Conflict of interest

Authors state no conflict of interest.

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